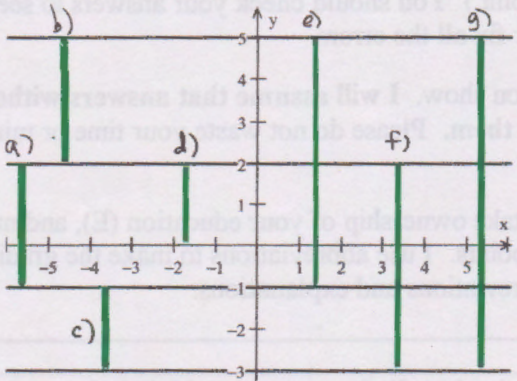
Math 250: Intro to 6.2, 6.3, and 6.4 (Area and Volume) "Find the length of the bar."

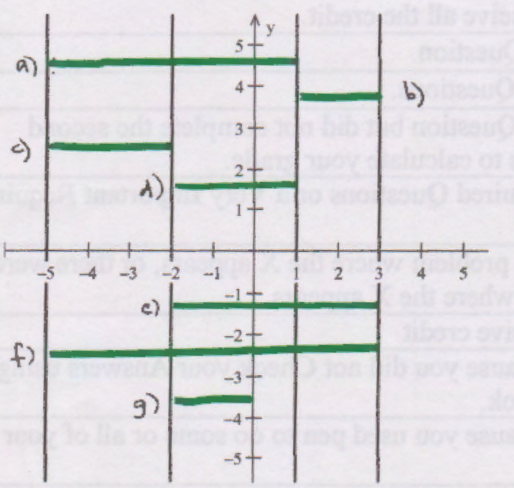
Note: There is no MML homework or textbook section for this skill, which will be needed throughout 6.2-6.4. Prepare for the PQ using this packet.

In general:

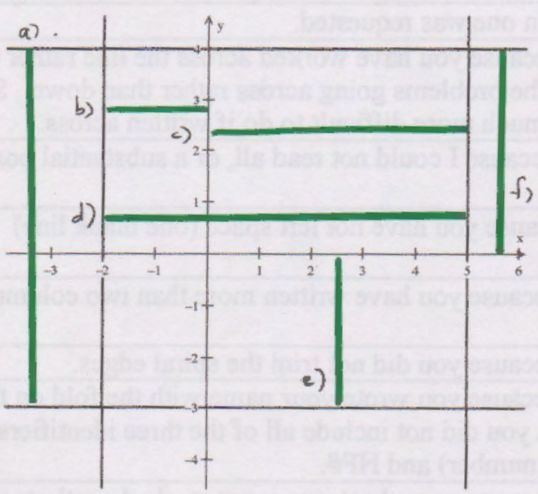
- When finding the length of a vertical bar, use (top y-coordinate) – (bottom y-coordinate).
- When find the length of a horizontal bar, use (right x-coordinate) – (left x-coordinate).



1)

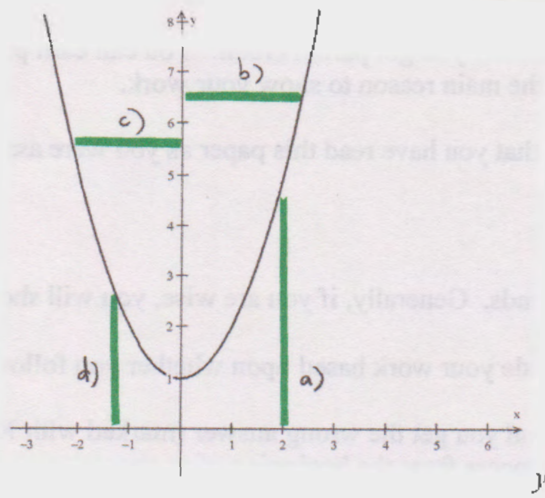


2)

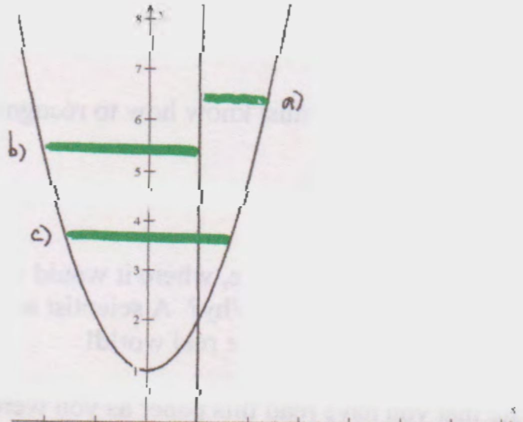


3)

Find the length of each bar i) in terms of x and ii) in terms of y.
~~If a vertical bar~~, for what values of x is the expression a valid length?
~~If a horizontal bar~~, for what values of y is the expression a valid length?
·Delete



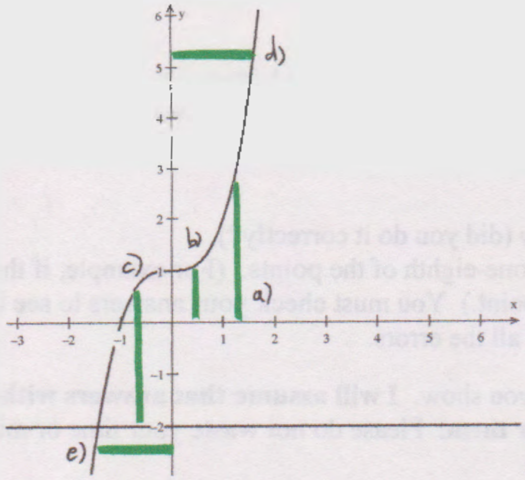
4) $y = x^2 + 1$



5) $y = x^2 + 1$

$x = 1$

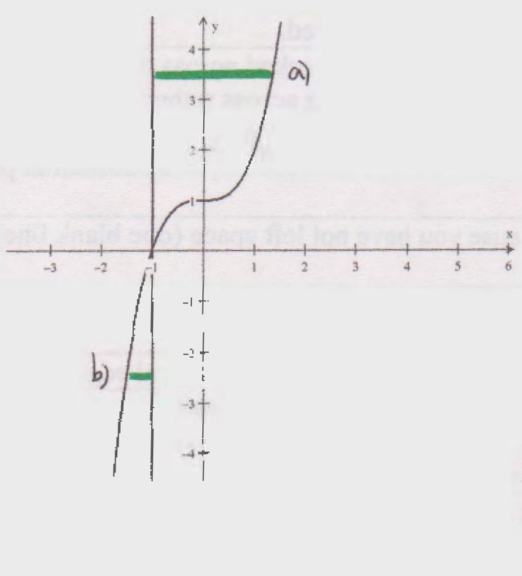
6)



$y = x^3 + 1$

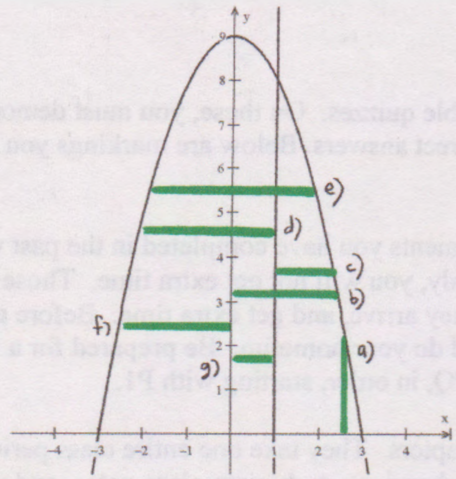
$x = -3$

7)



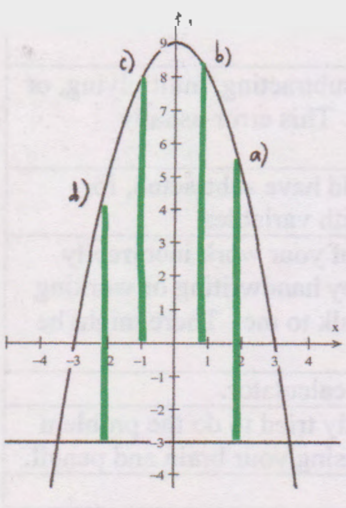
$y = x^3 + 1$

$x = -1$

8) 

$y = 9 - x^2$

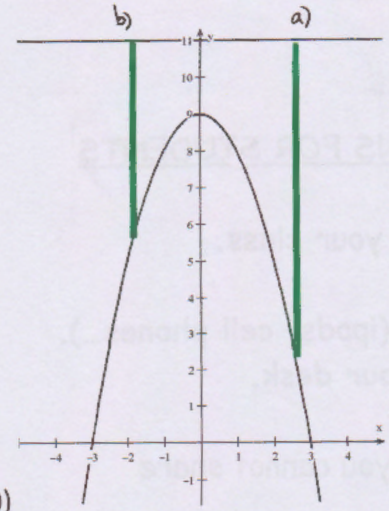
$x = 1$

9) 

$y = 9 - x^2$

$y = -3$

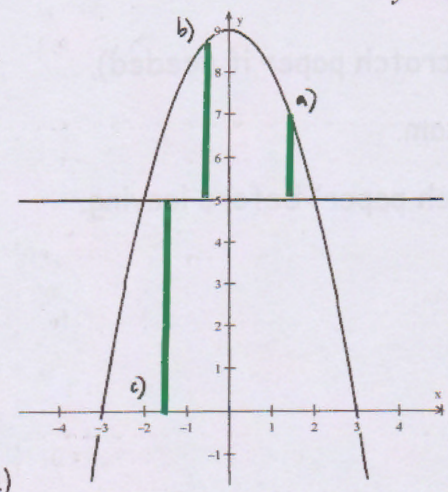
10)



$y = 9 - x^2$

$y = 11$

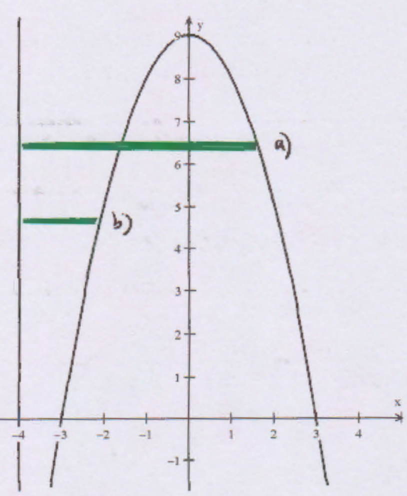
11)



$y = 9 - x^2$

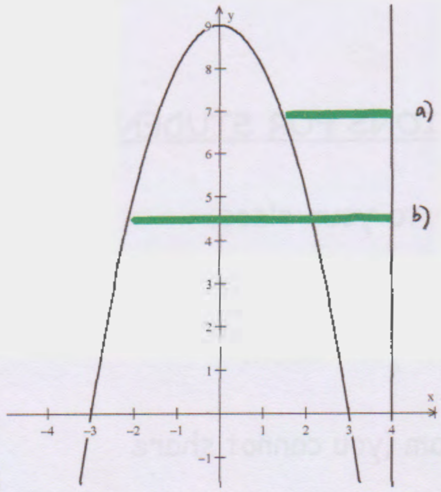
$y = 5$

12)



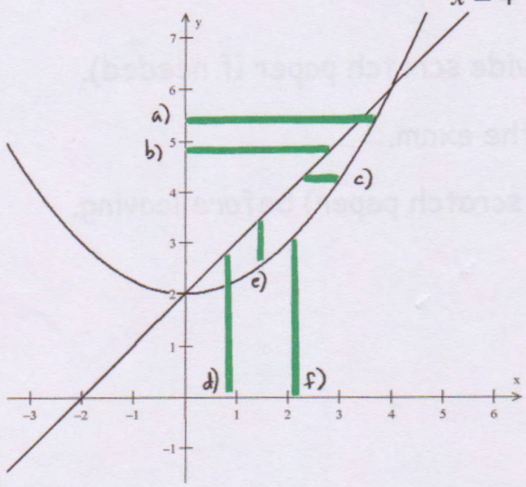
$y = 9 - x^2$

$x = -4$



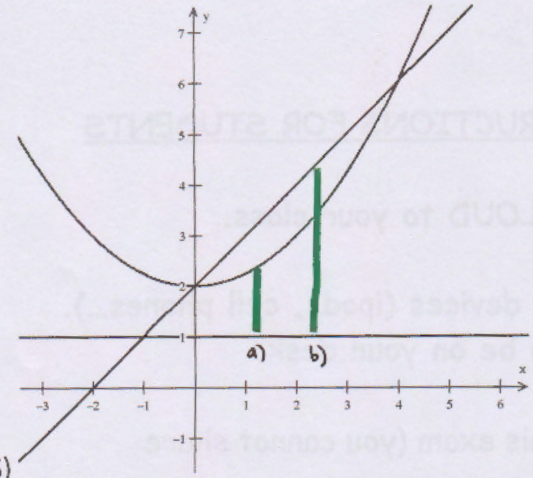
13) $y = 9 - x^2$

$x = 4$

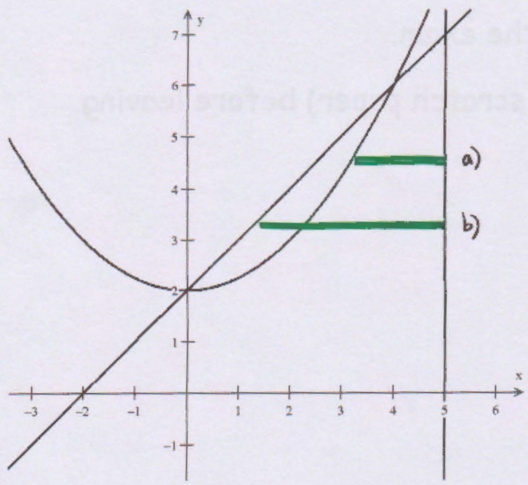


14) $y = \frac{1}{4}x^2 + 2$

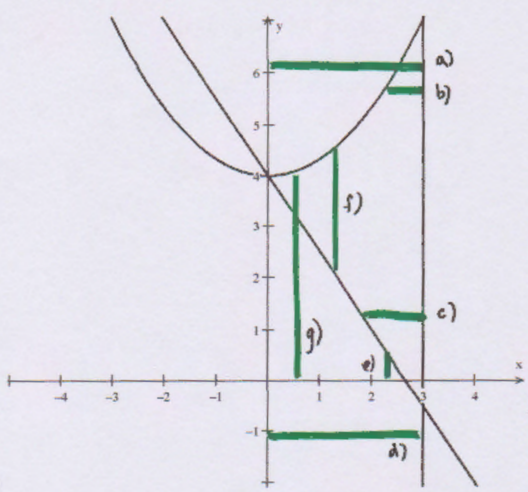
$y = x + 2$

15) 

$$y=\frac{1}{4}x^2+2$$
$$y=x+2$$
$$y=1$$

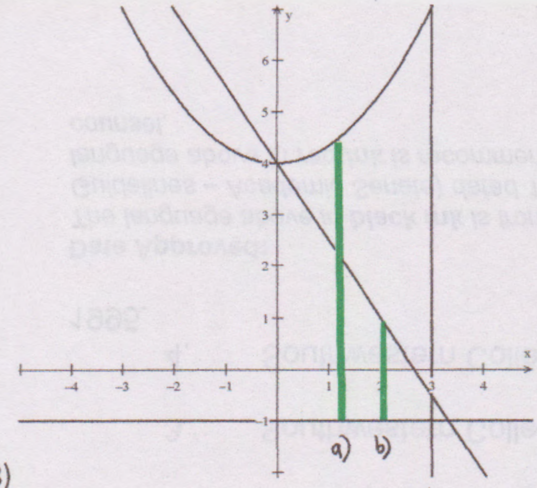
16) 

$$y=\frac{1}{4}x^2+2$$
$$y=x+2$$
$$x=5$$

17) 

$$y=\frac{1}{3}x^2+4$$
$$y=-\frac{3}{2}x+4$$
$$x=3$$

18)



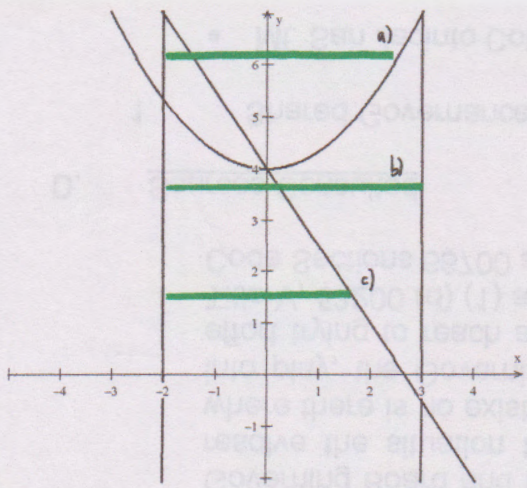
$$y=\frac{1}{3}x^2+4$$

$$y=-\frac{3}{2}x+4$$

$$x=3$$

$$y=-1$$

19)



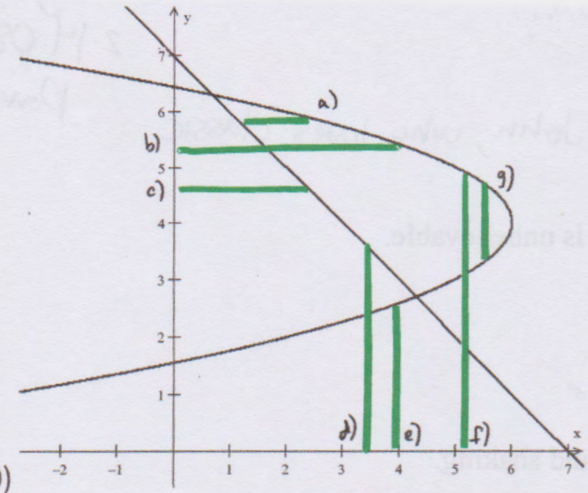
$$y=\frac{1}{3}x^2+4$$

$$y=-\frac{3}{2}x+4$$

$$x=3$$

$$x=-2$$

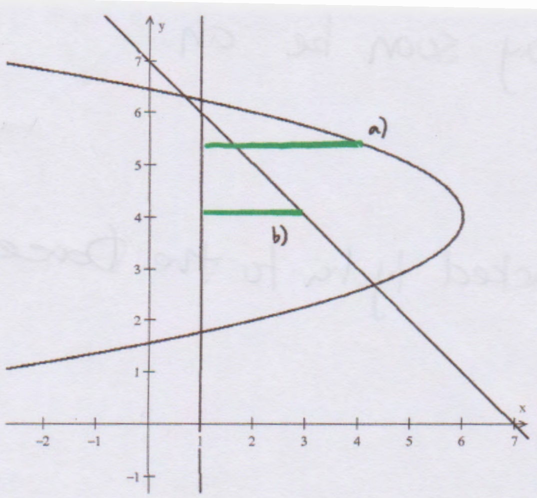
20)



$x = -(y-4)^2 + 6$

$x = 7 - y$

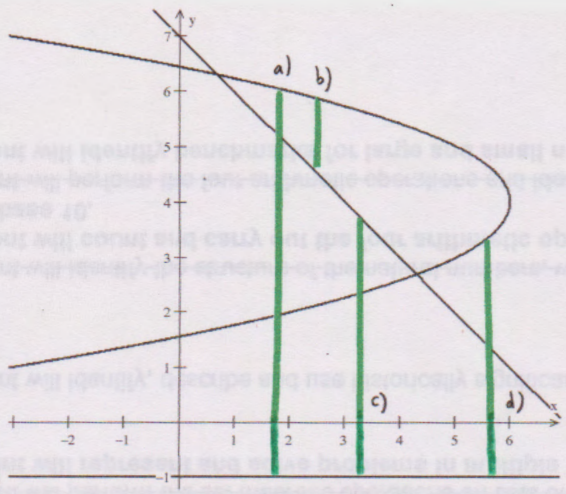
21)



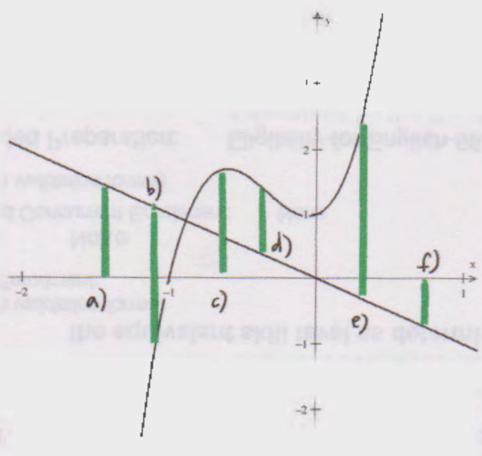
$x = 10 + 8y - y^2$

$x = 7 - y$

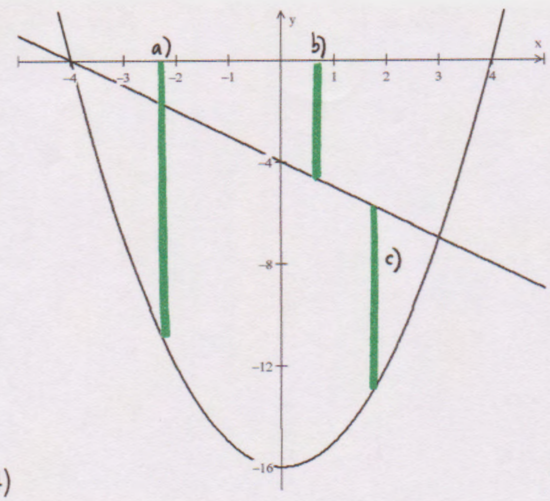
$x = 1$

22) 

$x = 10 + 8y - y^2$
$x = 7 - y$
$y = -1$

23) 

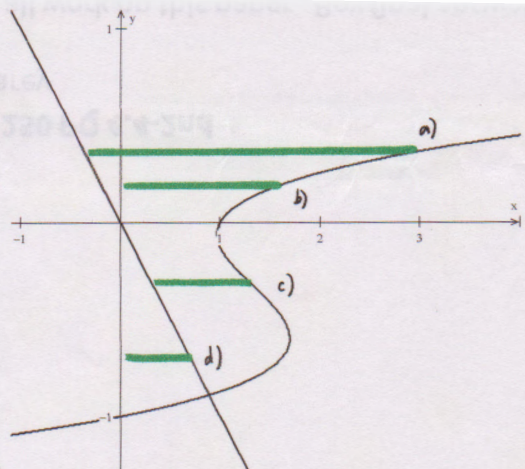
$f(x) = 9x^3 + 9x^2 + x + 1$
$g(x) = -x$



24)

$y = x^2 - 16$

$y = -x - 4$



25)

$f(y) = 9y^3 + 9y^2 + y + 1$

$g(y) = -y$

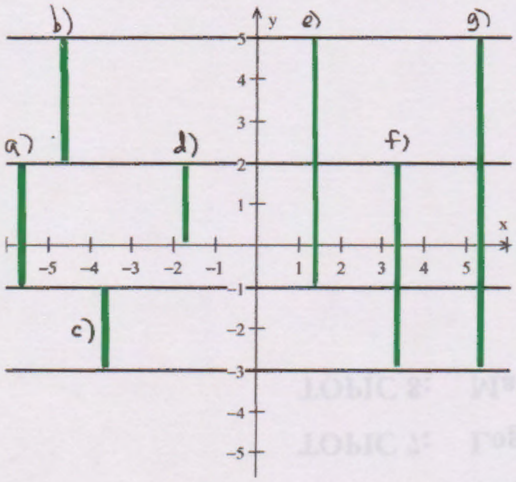
Math 250: Intro to 6.2, 6.3, and 6.4 (Area and Volume) "Find the length of the bar."

Note: There is no MML homework or textbook section for this skill, which will be needed throughout 6.2-6.4. Prepare for the PQ using this packet.

In general: Results are true lengths — should give positive results for all x, y!

- When finding the length of a vertical bar, use (top y-coordinate) – (bottom y-coordinate).
- When find the length of a horizontal bar, use (right x-coordinate) – (left x-coordinate).

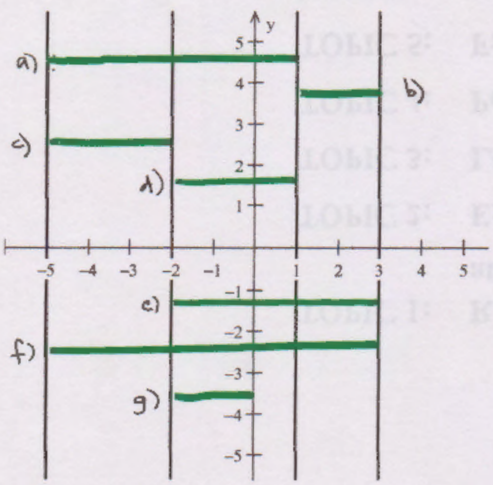
1)



a) $2-(-1) = \boxed{3}$
b) $5-2 = \boxed{3}$
c) $-1-(-3) = \boxed{2}$
d) $2-0 = \boxed{2}$
e) $5-(-1) = \boxed{6}$
f) $2-(-3) = \boxed{5}$
g) $5-(-3) = \boxed{8}$

all vertical bars
(top y) – (bottom y)

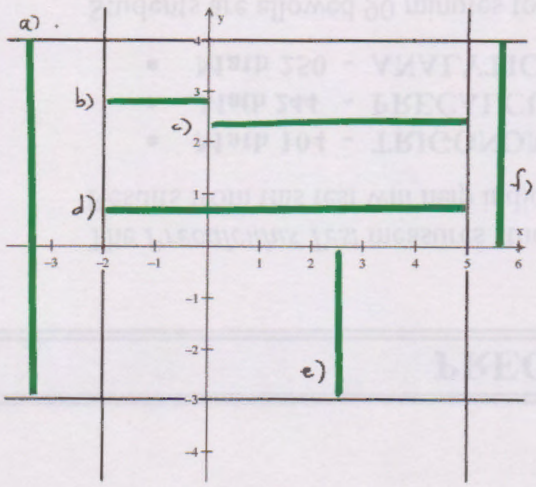
2)



a) $1-(-5) = \boxed{6}$
b) $3-1 = \boxed{2}$
c) $-2-(-5) = \boxed{3}$
d) $1-(-2) = \boxed{3}$
e) $3-(-2) = \boxed{5}$
f) $3-(-5) = \boxed{8}$
g) $0-(-2) = \boxed{2}$

all horizontal bars
(right x) – (left x)

3)

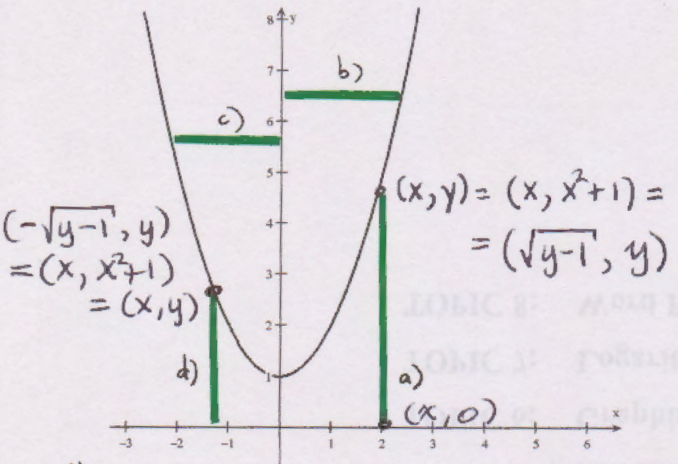


a) $4-(-3) = \boxed{7}$ vertical
b) $0-(-2) = \boxed{2}$ horizontal
c) $5-0 = \boxed{5}$ horizontal
d) $5-(-2) = \boxed{7}$ horizontal
e) $0-(-3) = \boxed{3}$ vertical
f) $4-0 = \boxed{4}$ vertical

Find the length of each bar i) in terms of x and ii) in terms of y.

for what values of x is the expression a valid length?

for what values of y is the expression a valid length?



4) $y = x^2+1$

Notice that an ordered pair can be expressed in terms of x as (x, x^2+1) as well as in terms of y.

$y = x^2+1$

$y-1 = x^2$

$\pm\sqrt{y-1} = x$

$x = \sqrt{y-1}$ is the right half

$x = -\sqrt{y-1}$ is the left half

• a) vertical (top y) − (bottom y)

i) in terms of x: $x^2+1-0 = \boxed{x^2+1}$ for all x, same as d) i)

ii) in terms of y: $y-0 = \boxed{y}$ for all y in range $(y \geq 1)$, same as d) ii)

b) horizontal (right x) − (left x)

i) in terms of x: $x - 0 = \boxed{x}$ for $x \geq 0$

ii) in terms of y: $\sqrt{y-1} - 0 = \boxed{\sqrt{y-1}}$ for all y in range $(y \geq 1)$ same as c) ii)

• c) horizontal

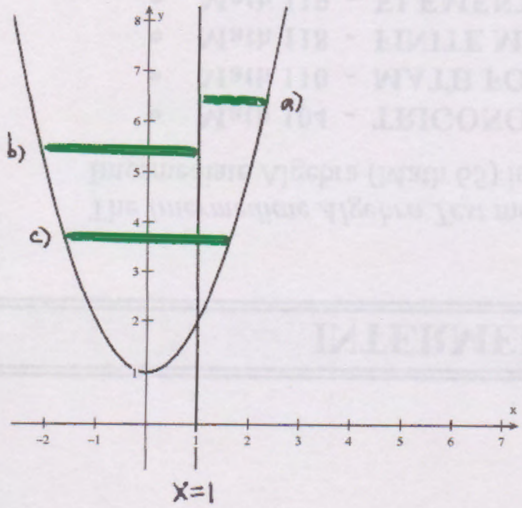
i) in x: $0 - x = \boxed{-x}$ for $x \leq 0$

ii) in y: $0 - (-\sqrt{y-1}) = \boxed{\sqrt{y-1}}$

d) vertical

i) in x: $(x^2+1) - 0 = \boxed{x^2+1}$

ii) in y: $y - 0 = \boxed{y}$



5) $y = x^2+1$

$x = 1$

a) horizontal

i) $\boxed{x-1}$ for $x \geq 1$

ii) $\boxed{\sqrt{y-1} - 1}$ for $y \geq 2$

b) horizontal

i) $\boxed{1-x}$ for $x \leq 1$

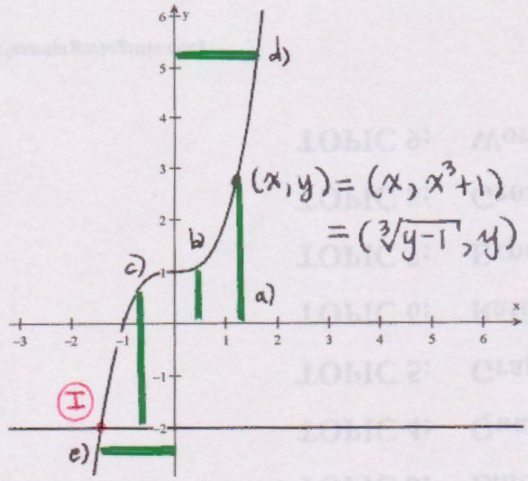
ii) $1 - (-\sqrt{y-1}) = \boxed{1+\sqrt{y-1}}$ for y in range $y \geq 1$

• c) horizontal

i) $x - (x) \Rightarrow$ nonsense!

ii) $\sqrt{y-1} - (-\sqrt{y-1}) = \boxed{2\sqrt{y-1}}$ in range, $y \geq 1$

$y = x^3 + 1$
$y - 1 = x^3$
$\sqrt[3]{y-1} = x$ No $\pm$ for odd-index radicals!

6) 

$y = x^3 + 1$
$x = -3$

a) vertical
i) $(x^3+1) - 0 = \boxed{x^3+1}$ $x \geq -1$
ii) $y - 0 = \boxed{y}$ $y \geq 0$

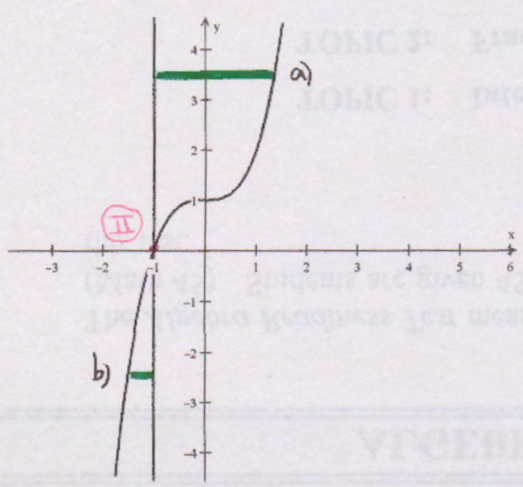
b) vertical; both i) and ii) same result as a)

• c) vertical
i) $(x^3+1) - (-2) = \boxed{x^3+3}$ $x \geq -\sqrt[3]{3}$
ii) $y - (-2) = \boxed{y+2}$ $y \geq -2$

intersection: I
$x^3 + 1 = -2$
$x^3 = -3$
$x = \sqrt[3]{-3} = -\sqrt[3]{3}$

d) horizontal
i) $x - 0 = \boxed{x}$ $x \geq 0$
ii) $\sqrt[3]{y-1} - 0 = \boxed{\sqrt[3]{y-1}}$ $y \geq 1$

• e) horizontal
i) $0 - x = \boxed{-x}$ $x \leq 0$
ii) $0 - \sqrt[3]{y-1} = \boxed{-\sqrt[3]{y-1}}$ $y \leq 1$



• a) horizontal
i) $x - (-1) = \boxed{x+1}$ $x \geq -1$
ii) $\sqrt[3]{y-1} - (-1) = \boxed{\sqrt[3]{y-1} + 1}$ $y \geq 0$

• b) horizontal
i) $\boxed{-1-x}$ $x \leq -1$
ii) $\boxed{-1-\sqrt[3]{y-1}}$ $y \leq 0$

intersection: II
$(-1, 0)$

7)
$y = x^3 + 1$
$x = -1$

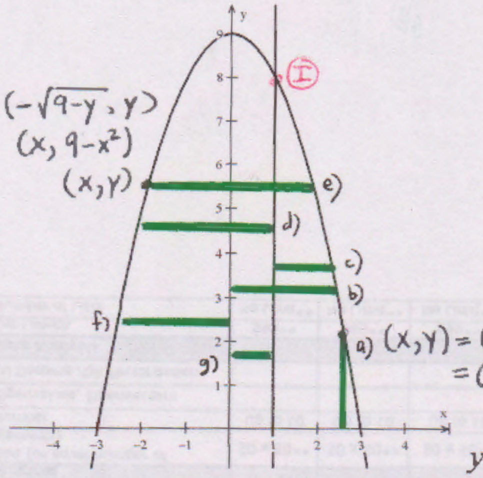
$y = 9 - x^2$

$x^2 = 9 - y$

$x = \pm\sqrt{9-y}$

$x = +\sqrt{9-y}$ (+) x coords $\Rightarrow$ right half

$x = -\sqrt{9-y}$ (−) x coords $\Rightarrow$ left half

8) 

$y = 9 - x^2$

$x = 1$

a) vertical

i) $9 - x^2 - 0 = \boxed{9-x^2}$ $\quad -3 \le x \le 3$

ii) $y - 0 = \boxed{y}$ $\quad 0 \le y \le 9$

b) horizontal

i) $x - 0 = \boxed{x}$ $\quad 0 \le x \le 3$

ii) $\sqrt{9-y} - 0 = \boxed{\sqrt{9-y}}$ $\quad 0 \le y \le 9$

c) horizontal

i) $\boxed{x-1}$ $\quad 1 \le x \le 3$

ii) $\boxed{\sqrt{9-y} - 1}$ $\quad 0 \le y \le 8$

intersection: I

$x = 1 \quad y = 9 - 1^2 = 8 \quad (1, 8)$

d) horizontal

i) $\boxed{1-x}$ $\quad -3 \le x \le 1$

ii) $1 - (-\sqrt{9-y}) = \boxed{1 + \sqrt{9-y}}$ $\quad 0 \le y \le 9$

e) horizontal

i) nonsense!

ii) $\sqrt{9-y} - (-\sqrt{9-y})$

$= \sqrt{9-y} + \sqrt{9-y} = \boxed{2\sqrt{9-y}}$ $\quad 0 \le y \le 9$

intersection: II & III

$-3 = 9 - x^2$

$x^2 = 9 + 3$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

II $(-2\sqrt{3}, -3)$ and $(2\sqrt{3}, -3)$ III

f) horizontal

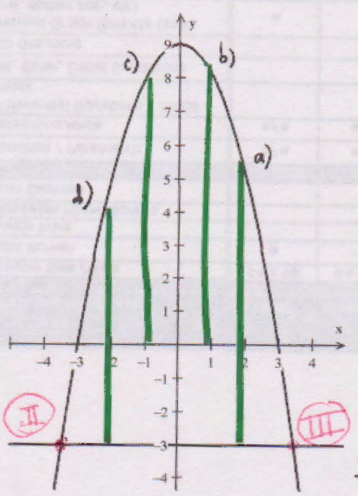
i) $0 - x = \boxed{-x}$ $\quad -3 \le x \le 0$

ii) $0 - (-\sqrt{9-y}) = \boxed{\sqrt{9-y}}$ $\quad 0 \le y \le 9$

g) horizontal

i) $1 - 0 = \boxed{1}$ $\quad$ any real x

ii) $= \boxed{1}$ $\quad$ any real y

9) 

$y = 9 - x^2$

$y = -3$

• a) i) $9 - x^2 - (-3) = 9 - x^2 + 3 = \boxed{12 - x^2}$ $\quad -2\sqrt{3} \le x \le 2\sqrt{3}$

ii) $y - (-3) = \boxed{y+3}$ $\quad -3 \le y \le 9$

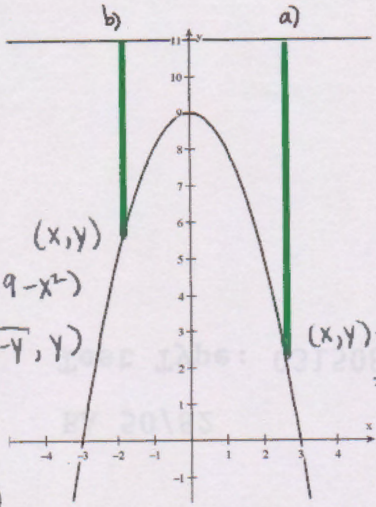
• b) i) $9 - x^2 - 0 = \boxed{9 - x^2}$ $\quad -3 \le x \le 3$

ii) $y - 0 = \boxed{y}$ $\quad -3 \le y \le 9$

c) same as a)

d) same as b)

all vertical lines



10) $y=9-x^2$

$y=11$

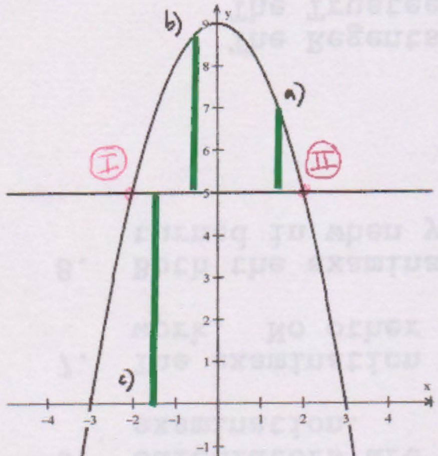
• a) vertical

i) $11-(9-x^2) = 11-9+x^2$

$= \boxed{x^2+2}$ all real x

ii) $\boxed{11-y}$ $y \le 9$

b) same as a)



11) $y=9-x^2$

$y=5$

a) vertical

i) $9-x^2-5 = \boxed{4-x^2}$ $-2 \le x \le 2$

ii) $\boxed{y-5}$ $5 \le y \le 9$

Intersection I & II

$9-x^2=5$

$4=x^2$

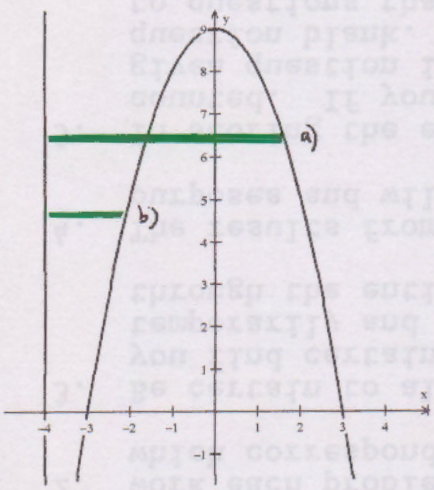
$\pm 2 = x$

b) same as a)

c) vertical

i) $5-0=\boxed{5}$ $-3 \le x \le 3$

ii) $5-0=\boxed{5}$ $0 \le y \le 5$



12) $y=9-x^2$

$x=-4$

Intersection III

off the graph $x=-4$

$y=9-(-4)^2$

$y=9-16$

$y=-7$

• a) horizontal

i) $x-(-4) = \boxed{x+4}$ $x \ge -4$

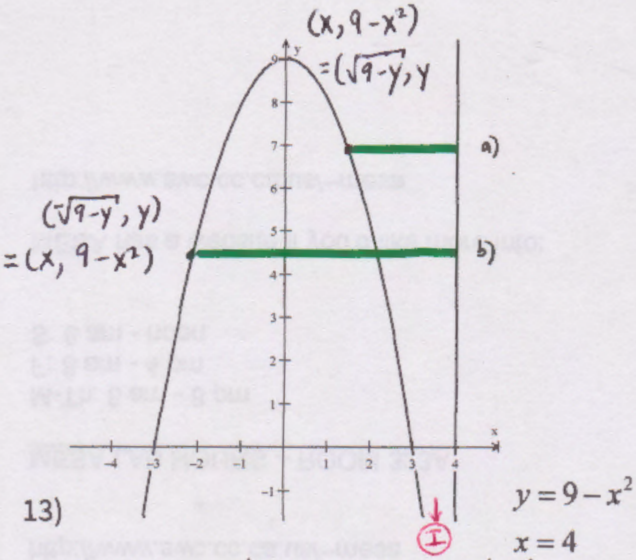
ii) $\sqrt{9-y}-(-4) = \boxed{\sqrt{9-y}+4}$ $-7 \le y \le 9$

• b) horizontal

i) $x-(-4) = \boxed{x+4}$ $x \ge -4$ same as a)

ii) $-\sqrt{9-y}-(-4) = \boxed{-\sqrt{9-y}+4}$ $-7 \le y \le 9$

13)



a) horizontal

i) $\boxed{4-x}$ $\quad x \le 4$

ii) $4-(+\sqrt{9-y}) = \boxed{4-\sqrt{9-y}}$ $\quad -7 \le y \le 9$

I intersection $9-4^2 = 9-16 = -7$

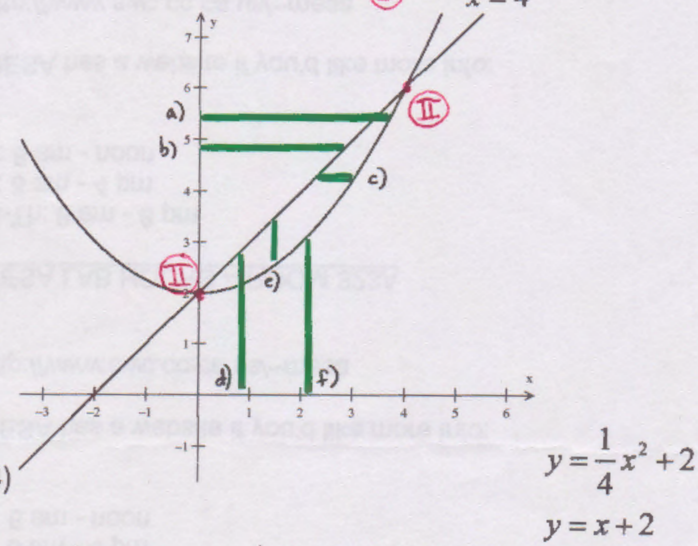
$(4, -7)$

b) horizontal

i) $\boxed{4-x}$ $\quad x \le 4$

ii) $4-(-\sqrt{9-y}) = \boxed{4+\sqrt{9-y}}$ $\quad y \le 9.$

14)



parabola $(x, \frac{1}{4}x^2+2)$

$\pm\sqrt{4(y-2)} = x$

$\pm\sqrt{4y-8}$ or $\pm 2\sqrt{y-2} = x$ $\Big\}$ $(2\sqrt{y-2}, y)$ right or $(-2\sqrt{y-2}, y)$ left

line $(x, x+2) = ((y-2), y)$

II intersection $\frac{1}{4}x^2+2 = x+2$

$\frac{1}{4}x^2 - x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, 4$

$(0,2)$ and $(4,6)$

a) horizontal

i) $x - 0 = \boxed{x}$ $\quad x \ge 0$

ii) $2\sqrt{y-2} - 0 = \boxed{2\sqrt{y-2}}$ $\quad y \ge 2$

b) horizontal

i) $x - 0 = \boxed{x}$ $\quad x \ge 0$

ii) $y-2-0 = \boxed{y-2}$ $\quad y \ge 2$

c) horizontal

i) $x - x$ nonsense

ii) $2\sqrt{y-2} - (y-2)$

$= \boxed{2\sqrt{y-2} - y + 2}$ $\quad y \ge 2$

d) vertical

i) $x+2-0 = \boxed{x+2}$ $\quad x \ge -2$

ii) $y - 0 = \boxed{y}$ $\quad y \ge 0.$

e) vertical

i) $x+2-(\frac{1}{4}x^2+2)$

$= x+2-\frac{1}{4}x^2-2$

$= \boxed{x - \frac{1}{4}x^2}$ $\quad 0 \le x \le 4$ II

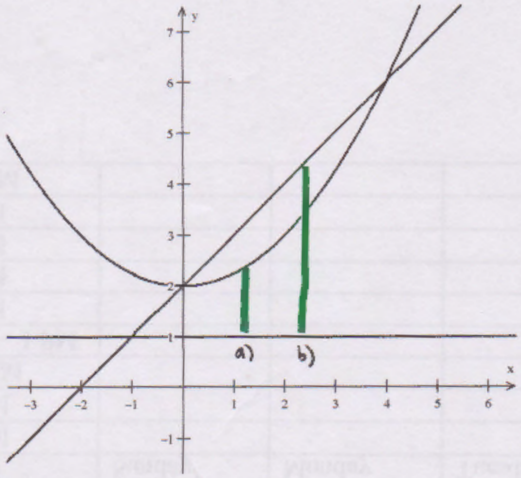
ii) $y - y$ nonsense!

f) vertical

i) $\frac{1}{4}x^2+2-0 = \boxed{\frac{1}{4}x^2+2}$ any x

ii) $y - 0 = \boxed{y}$ $\quad y \ge 2$

15)



$y=\frac{1}{4}x^2+2$

$y=x+2$

$y=1$

a) vertical

i) $\frac{1}{4}x^2+2-1=\boxed{\frac{1}{4}x^2+1}$ all x

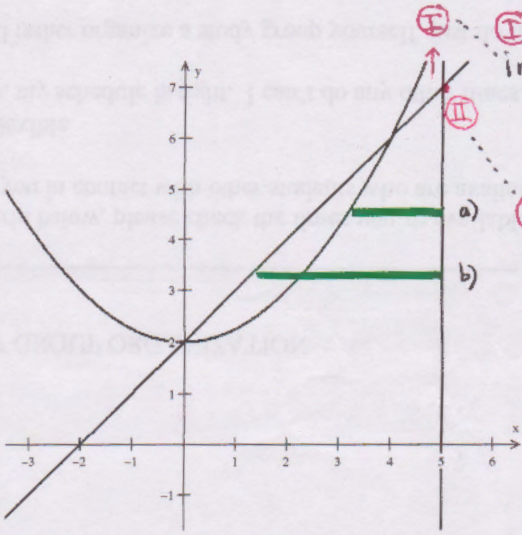
ii) $\boxed{y-1}$ $y\ge 1$

b) vertical

i) $x+2-1=\boxed{x+1}$ all x

ii) $\boxed{y-1}$ $y\ge 1$

16)



$y=\frac{1}{4}x^2+2$

$y=x+2$

$x=5$

I intersection

$\frac{1}{4}5^2+2=\frac{33}{4}$

$(5,\frac{33}{4})$

II $5+2=7$

$(5,7)$

a) horizontal

i) $5-(\frac{1}{4}x^2+2)$

$=5-\frac{1}{4}x^2-2$

$=\boxed{3-\frac{1}{4}x^2}$ $x\le 5$

ii) $\boxed{5-2\sqrt{y-2}}$ $2\le y\le \frac{33}{4}$ I

b) horizontal

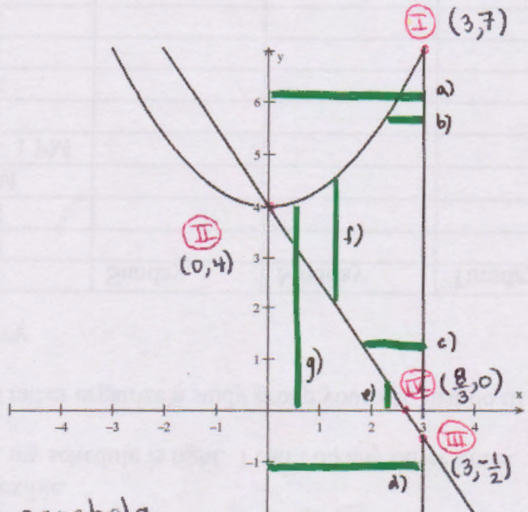
i) $\boxed{5-x}$ $x\le 5$

ii) $5-(y-2)$

$=5-y+2$

$=\boxed{7-y}$ $y\le 7$ II

17) parabola



$y=\frac{1}{3}x^2+4$

$y=-\frac{3}{2}x+4$

$x=3$

$(x,\frac{1}{3}x^2+4)=(\sqrt{3(y-4)},y)$ right

$=(-\sqrt{3(y-4)},y)$ left

line $(x,-\frac{3}{2}x+4)=(-\frac{2}{3}y+\frac{8}{3},y)$

a) $3-0=\boxed{3}$ any x, any y

b) i) $\boxed{3-x}$ $x\le 3$

ii) $\boxed{3-\sqrt{3(y-4)}}$ $4\le y\le 7$ I

c) i) $\boxed{3-x}$ $x\le 3$

ii) $3-(-\frac{2}{3}y+\frac{8}{3})=\boxed{\frac{2}{3}y+\frac{1}{3}}$ III II $-\frac{1}{2}\le y\le 4$

d) same as a)

e) i) $-\frac{3}{2}x+4-0=\boxed{-\frac{3}{2}x+4}$ $x\le\frac{8}{3}$ IV

ii) $y-0=\boxed{y}$ $y\ge 0$

f) i) $\frac{1}{3}x^2-4-(-\frac{3}{2}x+4)$

$=\boxed{\frac{1}{3}x^2+\frac{3}{2}x-8}$ II I, III $0\le x\le 3$

ii) $y-y$ nonsense

g) i) $\frac{1}{3}x^2+4-0=\boxed{\frac{1}{3}x^2+4}$ any x

ii) $y-0=y$ $\boxed{y\ge 4}$

Intersection

$-1 = -\frac{3}{2}x + 4$

$-5 \cdot \left(-\frac{2}{3}\right) = x$

$\frac{10}{3} = x$

$\left(\frac{10}{3}, -1\right)$

a) i) $\frac{1}{3}x^2 + 4 - (-1)$

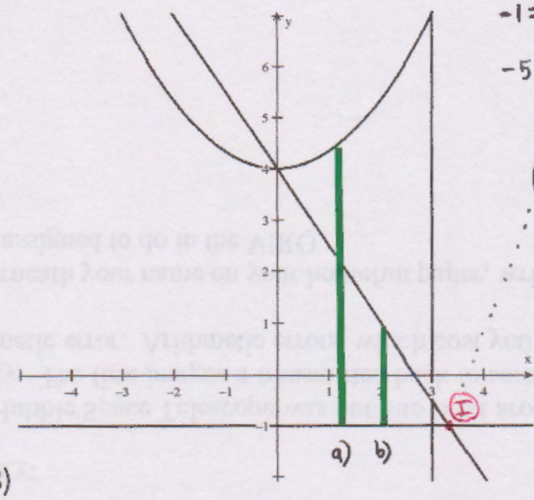
$= \boxed{\frac{1}{3}x^2 + 5}$ any x

ii) $y - (-1) = \boxed{y+1}$ $y \geq 4$

b) i) $-\frac{3}{2}x + 4 - (-1)$

$= \boxed{-\frac{3}{2}x + 5}$ $x \leq \frac{10}{3}$

ii) $y - (-1) = \boxed{y+1}$ $y \geq -1$

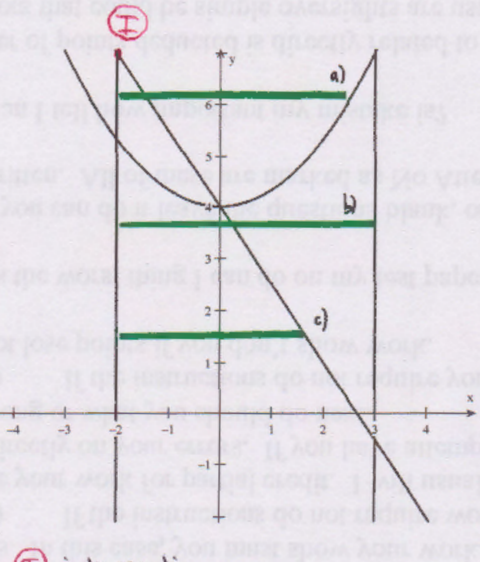


18) $y = \frac{1}{3}x^2 + 4$

$y = -\frac{3}{2}x + 4$

$x = 3$

$y = -1$



a) i) $x - (-2) = \boxed{x+2}$ $x \geq -2$

ii) $\sqrt{3(y-4)} - (-2)$

$= \boxed{\sqrt{3(y-4)} + 2}$ $y \geq 4$

b) $3 - (-2) = \boxed{5}$ any x, any y

c) i) $x - (-2) = \boxed{x+2}$ $x \geq -2$

ii) $-\frac{2}{3}y + \frac{8}{3} - (-2)$

$= \boxed{-\frac{2}{3}y + \frac{14}{3}}$ $y \leq 7$ I

19) $y = \frac{1}{3}x^2 + 4$

$y = -\frac{3}{2}x + 4$

$x = 3$

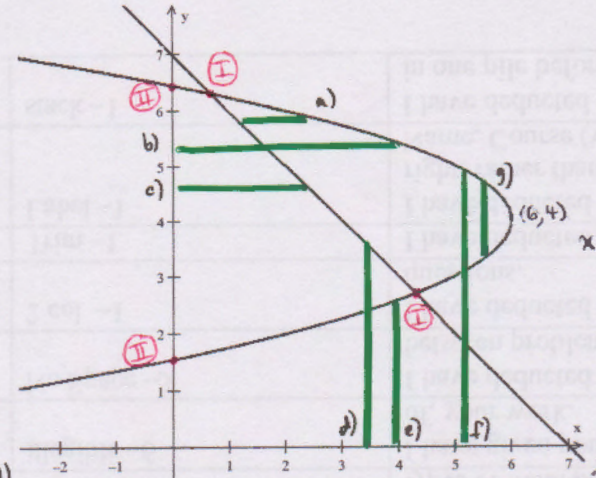
$x = -2$

I intersection

$y = -\frac{3}{2}(-2) + 4$

$y = 3 + 4 = 7$

$(-2, 7)$



20)

$x = -y^2 + 8y - 10$

$-y^2 + 8y - 16 + 6$

$-(y^2 - 8y + 16) + 6$

$x = -(y-4)^2 + 6$

$x = 7 - y$

parabola $(-y^2 + 8y - 10, y)$

$y = 4 \pm \sqrt{-x+6}$

$= (x, 4 + \sqrt{6-x})$ top half

$= (x, 4 - \sqrt{6-x})$ bottom half

line $(7-y, y)$

$= (x, 7-x)$

I intersections $-y^2 + 8y - 10 = 7 - y$

$0 = y^2 - 9y + 17$

$y = \frac{9 \pm \sqrt{13}}{2}$

II y-int $0 = -(y-4)^2 + 6$

$4 \pm \sqrt{6} = y$

a) i) $x - x$ nonsense

ii) $-y^2 + 8y - 10 - (7 - x)$

$= \boxed{-y^2 + 9y - 17}$ I $\frac{9-\sqrt{13}}{2} \le y \le \frac{9+\sqrt{13}}{2}$

b) i) $x - 0 = \boxed{x}$ $x \ge 0$

ii) $-y^2 + 8y - 10 - 0 = \boxed{-y^2 + 8y - 10}$

II $4 - \sqrt{6} \le y \le 4 + \sqrt{6}$

c) i) $x - 0 = \boxed{x}$ $x \ge 0$

ii) $7 - y - 0 = \boxed{7 - y}$ $y \le 7$

d) i) $y - 0 = \boxed{y}$ $y \ge 0$

ii) $7 - x - 0 = \boxed{7 - x}$ $x \le 7$

e) i) $4 - \sqrt{6-x} - 0 = \boxed{4 - \sqrt{6-x}}$ $x \le 6$

ii) $y - 0 = \boxed{y}$ $y \le 4$

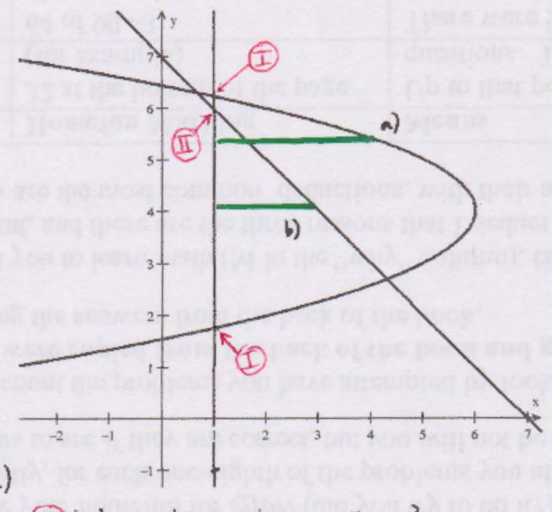
f) i) $4 + \sqrt{6-x} - 0 = \boxed{4 + \sqrt{6-x}}$ $x \le 6$

ii) $y - 0 = \boxed{y}$ $y \ge 4$

g) i) $4 + \sqrt{6-x} - (4 - \sqrt{6-x})$

$= \boxed{2\sqrt{6-x}}$ $x \le 6$

ii) $y - y$ nonsense



21)

$x = 10 + 8y - y^2$

$x = 7 - y$

$x = 1$

I intersection $1 = -10 + 8y - y^2$

$y^2 - 8y + 11 = 0$

$y = 4 \pm \sqrt{5}$

II intersection $1 = 7 - y$

$y = 6$ $(1, 6)$

a) i) $\boxed{x - 1}$ $x \ge 1$

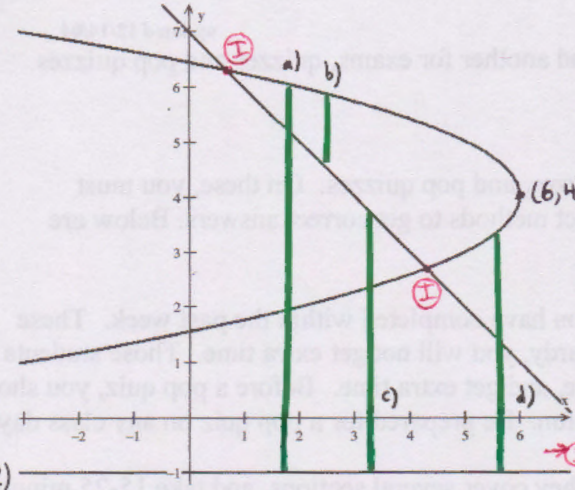
ii) $-y^2 + 8y - 10 - 1$

$= \boxed{-y^2 + 8y - 11}$ $4 - \sqrt{5} \le y \le 4 + \sqrt{5}$

b) i) $\boxed{x - 1}$ $x \ge 1$

should be same as above!

ii) $7 - y - 1 = \boxed{6 - y}$ $y \le 6$



22) $x=10+8y-y^2$

$x=7-y$

$y=-1$

should be same as 20 and 21

a) i) $4+\sqrt{6-x}-(-1)=\boxed{5+\sqrt{6-x}}$ $x\le 6$

ii) $y-(-1)=\boxed{y+1}$ $y\ge -1$

b) i) $4+\sqrt{6-x}-(7-x)$ I

$=\boxed{-3+\sqrt{6-x}+x}$ $\frac{5}{2}-\frac{\sqrt{13}}{2}\le x\le \frac{5}{2}+\frac{\sqrt{13}}{2}$

ii) y-y nonsense

c) i) $7-x-(-1)=\boxed{8-x}$ $x\le 8$ II

ii) $y-(-1)=\boxed{y+1}$ $y\ge -1$

d) i) $4-\sqrt{6-x}-(-1)=\boxed{5-\sqrt{6-x}}$ $x\le 6$

ii) $y-(-1)=\boxed{y+1}$ $y\ge -1$

I see 20 $y=\frac{9\pm\sqrt{13}}{2}$

$x=-10+8\left(\frac{9\pm\sqrt{13}}{2}\right)-\left(\frac{9\pm\sqrt{13}}{2}\right)^2$

or

$x=-\left(\frac{9\pm\sqrt{13}}{2}-4\right)^2+6$

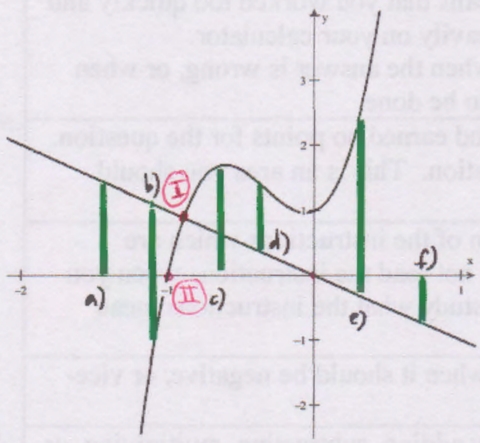
scratch paper ...→ $x=\frac{5}{2}\pm\frac{\sqrt{13}}{2}$

II Intersection

$x=7-(-1)$

$x=8$

$(8,-1)$



23) $f(x)=9x^3+9x^2+x+1$

$g(x)=-x$

a) $-x-0=\boxed{-x}$ $x\le 0$

b) $-x-(9x^3+9x^2+x+1)$

$=-x-9x^3-9x^2-x-1$

$=\boxed{-9x^3-9x^2-2x-1}$

$x\le -.89057$ I

c) $9x^3+9x^2+x+1-0$

$=\boxed{9x^3+9x^2+x+1}$ $x\ge -1$ II

d) $9x^3+9x^2+x+1-(-x)$

$=\boxed{9x^3+9x^2+2x+1}$ $x\ge -.89057$ I

e) same as d)

f) $0-(-x)=\boxed{x}$ $x\ge 0$

* cannot solve f(x) for x (easily) in terms of y

Only do these bars in terms of x.

↳ intersection: approximated for the same reason.

I $9x^3+9x^2+x+1=-x$

$9x^3+9x^2+2x+1=0$

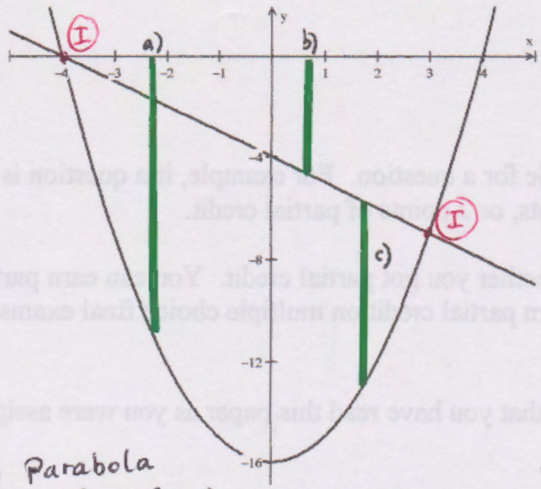
$x\approx -.89057$

II $9x^3+9x^2+x+1=0$

$9x^2(x+1)+1(x+1)=0$

$(9x^2+1)(x+1)=0$

$x=-1$



$a)\ i)\ 0-(x^2-16)=\boxed{-x^2+16}\quad -4\le x\le 4$

$ii)\ 0-y=\boxed{-y}\quad y\le 0$

$b)\ i)\ 0-(-x-4)$

$=\boxed{x+4}\quad x\ge 4$

$ii)\ 0-y=\boxed{-y}\quad y\le 0$

$c)\ i)\ -x-4-(x^2-16)$

$=\boxed{-x^2-x+12}\quad -4\le x\le 3$

$ii)\ y-y$ nonsense

24) Parabola

(x, x^2-16)

$(\sqrt{y+16}, y)$ right half

$(-\sqrt{y+16}, y)$ left half

Line $(x, -x-4)$

$(-y-4, y)$

$y=x^2-16$

$y=-x-4$

I intersection

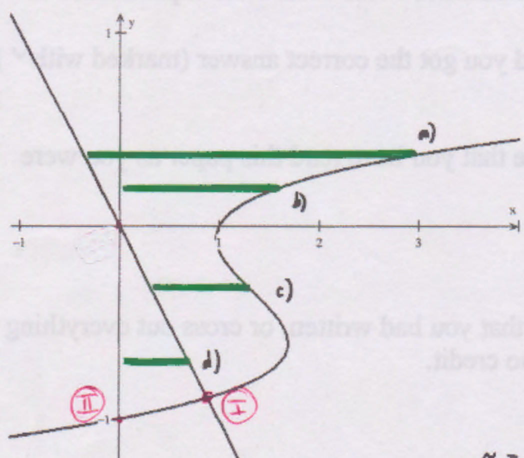
$x^2-16=-x-4$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4, 3$

$(-4,0)$ and $(3,-7)$



25)

$x=f(y)=9y^3+9y^2+y+1$

$x=g(y)=-y$

$a)\ 9y^3+9y^2+y+1-(-y)$

$=\boxed{9y^3+9y^2+2y+1}\quad y\ge -.89$ I

$b)\ 9y^3+9y^2+y+1-0$

$=\boxed{9y^3+9y^2+y+1}\quad y\ge -1$ II

c) same as a)

$d)\ -y-0=\boxed{-y}\quad y\le 0.$

* cannot solve $f(y)$ for y (easily) in terms of x

Only do these bars in terms of y.

↳ intersection approximated for the same reason

Note: intersections mirror answers in 23

I $y\approx -.89057$

II $y=-1$